An approach of pairing bevel gears from conventional cutting machine with gears produced on 5-axis milling machine

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ABSTRACT

The authors developed a new method to automatically find the optimal topological modification from the predetermined measurement grid points for bevel gears. By using the method, it is possible to duplicate any flank form of a bevel gear given by the measurement points and to provide the 3D model for CAM machining in a very short time. This method not only allows the user to model existing flank forms into 3D models, but also can be applied for various other purposes, such as compensating hardening distortions and manufacturing deviations which are very important issues but not yet solved in the practical milling process.

1 INTRODUCTION

Recently, the cutting bevel gears on universal 5-axis milling machines has been widely accepted as a promising solution to replace the conventional cutting process. The process is highly flexible and does not require special tools. Thus, it is particularly suitable for small batches, prototypes, repairs in use having unacceptably high lead times. In order to apply the milling process for bevel gear cutting, we should provide feasible solid models. However, the kinematic geometry of the bevel gears is relatively complicated in accordance with the variety of the cutting method such as Gleason (fixed settings, Duplex® and Zerol®), Klingelnberg (Cyclo-Palloid® and Palloid®) and Oerlikon and it's not easy to generate the 3D geometry model proper for milling. In the calculation software KISSsoft (1), the geometry calculation of straight and skew bevel gears for standard cone types has been available since many years in accordance with ISO 23509 (2). Then, the expansion to 3D models of spiral bevel gears was made covering all cone types four years ago. Since the 3D models of the spiral bevel gears are available, there has been much interest from many companies worldwide. The first prototype based on the 3D model from KISSsoft was machined by one of the major 5-axis milling machine manufacturers, Breton in Italy (3), and gave very satisfactory results. Then one of their customer who is using a 5-axis milling machine wanted to produce a very large bevel gear pair to replace an existing gear pair. However, they had a special problem hard to resolve. The problem was that the pinion shaft having 1500mm length was too long to be cut on the Breton machine. So the pinion was produced on a conventional Gleason machine, but the customer wanted to produce the gear (d_e2 = 500mm) on the Breton machine. We always recommend our users that the model for the pinion and gear must be generated by the same software and thus the combination of a pinion, manufactured on a Gleason machine should not be combined with a gear based on the model from KISSsoft. But the customer insisted, so we had to invent something!
We got the basic gear data and the measurement grid points of the flank form of the gear produced by their Gleason software from the customer. However, the design data didn’t include the formal definition of the flank modifications. Thus, the comparison of these measurement points with the 3D model from KISSsoft naturally showed small deviations. The deviation could not be eliminated easily by varying the geometric parameters and applying typical modifications such as barreling (profile crowning) and lead crowning. Thus, we developed a creative solution to generate a 3D model of the gear from KISSsoft and to adapt it to the given grid point from Gleason. In the following chapters, we will show the procedure of the method and the application results.

2 TOPOLOGICAL MODIFICATION OF THE 3D MODEL

The basic cone geometry of the bevel gear can be defined in accordance with ISO 23509, and the flank form is defined from the transverse tooth forms calculated along the face width. The trace form will be the extended epicycloid form by face hobbing process or circular form by face milling, as shown in Figure 1. In KISSsoft, the tooth form is supposed to the planar involutes of the virtual spur gear in transverse section. Then, the tooth flank surface is generated by splining the tooth forms of each section.

![Extended epicycloid and Circular](image)

Figure 1. Face hobbing (left) and face milling (right) processes

Bevel gear machine tool manufacturers (such as Klingelnberg and Gleason) have their own methods to generate the tooth form based on the generating motion of the cutter. The tooth form is known as octoid and is slightly different from spherical or planar involute tooth form. However, the difference of the tooth forms is normally less than the tolerance range and will have no problem in practical use. This can be verified from the fact that the bevel gears are always produced in pairs by the same process in order to achieve a good contact pattern in practice. In order to validate the practical usage of the 3D model from KISSsoft, we compared our model with reference models of manufacturer programs and also carried out the contact pattern check with the actual model. The result showed the tooth flanks along the face width of the two models are very well matched with only slight differences (4).

It’s one of the most important tasks to find the optimal modification to give good contact pattern in a bevel gear pair. In KISSsoft, the contact pattern of the bevel gear pair can be easily optimized by using proper modifications as shown in Figure 2. There are eight types of the modifications available for bevel gears in KISSsoft (profile crowning, eccentric profile crowning, pressure angle modification, helix angle modification, lead crowning, eccentric lead crowning, twist, and topological modification). The user can define different combination of modifications for drive and coast flanks to optimize the contact pattern separately.
However, if the target modification has highly non-linear or irregular pattern, the simple combination of the conventional modifications cannot be applied. In that case, the topological modification should be used to allow the user to freely define any type of modification that can't be covered by the conventional modifications. The user can define the modifications in a data map of factors at any position along the face width and along the tooth height by using the topological modification following the convention in ISO 21771 (5) as shown in Figure 3.

Figure 3. Definition of topological modification in ISO 21771 (5)
Figure 4 shows an example of the file structure of the modification used in KISSsoft. The example data map defines the progressive tip relief on side I and no modification on side II. Note that the modification values in the data map is normalized and the actual local modifications are calculated with \( Ca_{\text{local}} = f_{ij} \times Ca \), where \( f_{ij} \) is the modification factor at \((i, j)\) node and \( Ca \) is the amount of modification. The intermediate values in between the data can be interpolated by linear, quadratic, or spline approximation along the tooth width and height respectively.

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The adjustment of the bevel gear models to any predetermined measurement grid points should now be possible by applying the topological modification. That is, the modification can be calculated as the deviation between the surface of the 3D model from KISSsoft and the measurement grid points of the target model. The measurement grid points report contains the Cartesian coordinates and the normal vectors of the grid points with the format of [XP YP ZP XN YN ZN]. The reference coordinate system of the data is different according to the measurement machines. For example, the reference coordinate system of Klingelnberg format is using the convention shown in Figure 5. The order of the indexes for the points and the sections are defined according to ISO/TR 10064-6 (6) as well as the convention from the manufacturers such as Klingelnberg (7), here the index of the lines starts from the root to the tip, and the index of the columns from the side II (heel) to the side I (toe).
In applying the modification, however, various problems have shown up. The definition of topological modification surface in helical gears is located between the tip and the root form diameters, but the diameters over the tooth width for bevel gears are changing.
On the other hand, the effort to transform the measured grid points to the format of the topological modifications is greatly increased. While the measurement direction of the distance between the two corresponding grid points for adjustment calculation is different from the normal of the tooth form (that is, the path of contact) along which the modification is applied. Moreover, even the deviation values are given correctly, we cannot easily reach to the exact surface points because the target modification can have highly nonlinear pattern.

Thus, the procedure to get the topological modification, so that the final model becomes equivalent with the target model, cannot be finished in just a single step but need several iterations as shown in Figure 6. In each step, the distance between the corresponding measurement points are calculated and converted into the dimension in the virtual cylindrical gear. Then the topological modification is calculated based on these values and applied to generate a new measurement grid. The procedure iterates until the given acceptance criteria is met. The acceptance criteria is given as the maximum distance between the surface of the 3D model and the corresponding measurement points is smaller than the user-defined tolerance.

3 APPLICATION AND RESULT

We used 11x7 points for the measurement and topology template definition, that is, 11 points starting from the side I (toe) to side II (heel), and 7 points from the root form diameter to the tip diameter without any margins. The position of each measurement point is defined as the length factor of the path of contact from the root form diameter to the tip diameter (column values in yellow in Table 1) and the face width factor from the side I to the side II (row values in yellow in Table 1).

3.1 Topological modification for the right flank

Table 1 shows the topological deviation and modification template values for the right flank according to the calculation steps. In the calculation, we set the acceptable maximum deviation to 5μm.

Step 1
In the first step, we measure the deviation by the normal distance between the measurement points of the Gleason model with the flank surface of 3D model of KISSsoft (see Deviation 1 in Table 1). Then, we use the Deviation 1 as the initial topological template, Modification 1. The green-colored fields in the table indicates the border of the tooth flank. In our modeling strategy, we use slightly bigger surface area to cover the real gear surface, and it’s not possible to measure correct distances at the borders. Thus, we ignore the border values in the acceptance checking in the calculation procedure and use the extrapolated values for the values. The maximum distance of the initial step gives 575μm at the position (0.965, 0.696). The deviation shows relatively big values because we intentionally increased the tooth thickness of the KISSsoft model to completely cover the surface of the target model and to give positive distances. Thus, the final model is compensating not only the topological deviation of the surface but also the tooth thickness deviation of the model.

Step 2
After applying the topological modification of the first step, the maximum distance at the position (0.965, 0.696) reduced to 65μm and the new maximum distance is 135μm at the position (0, 0.879) (see Deviation 2 in Table 1). From the Deviation 2, you will see the three points at (0, 0.089), (0.522, 0.089) and (0.965, 0.193) have the deviation less than the acceptance criteria of 5μm (values in blue). In this case, we use the same topological modification values of the last step at those positions. For the rest positions,
we build a new topological modification by linear summation of the deviation of each point and the last topological modification, that is,

\[ \text{Modification}_2 = \text{Modification}_1 + \text{Deviation}_2. \]

**Step 3**
Now the Deviation 3 after applying the Modification 2 shows smaller distances than the Deviation 2 and more positions fitting into the acceptance deviation. The new maximum distance is 70\( \mu \text{m} \) at the position (0.965, 0.879) (see Deviation 2 in Table 1). However, the deviation in several positions, such as the positions at (0.956, 0.089) and (0.956, 0.193), increased because the surface is generated by spline approximation from the topological modification template (values in red). In this case, we build a new topological modification from the last topological modification, that is,

\[ \text{Modification}_3 = \text{Modification}_2 - \text{SIGN}(\text{Modification}_2 - \text{Modification}_1) \times (\text{Deviation}_2) + \left(\frac{\text{SIGN}(\text{Deviation}_2) + \text{SIGN}(\text{Deviation}_3)}{2}\right) \times (\text{Deviation}_2 - \text{Deviation}_3). \]

**Step 11 (Final step)**
Then, we needed to iterate 11 steps until all the deviations fitting into the acceptance criteria. You can find the final topological modification as Modification 11 and the final deviation as Deviation 12 in Table 1. Now all the deviation values are less than the maximum deviation of 5\( \mu \text{m} \) except the values at the border. The graphical comparison of the modification surfaces of Step 1 and the Step 11 (final step) are shown in Figure 1. As you can expect, the final modification surface doesn't not show regular pattern, and it's impossible to achieve the modification by simple combination of the conventional modification types such as crowning and barrelling.

![Figure 7. Modifications for right flank at Step 1 (left) and Step 11 (right)](image)

### 3.2 Topological modification for the left flank
After finishing the calculation for the right flank, we applied the same procedure for the left flank. Table 2 shows the topological deviation and modification template values according to the calculation steps for the left flank.

**Step 1**
In the first step, the maximum distance of the left flank shows 570\( \mu \text{m} \) at the position (0.965, 0.789).
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| Final step | 2 | 1  | 376 | 432 | 498 | 555 | 602 | 645 | 675 | 698 | 713 | 726 | 744 |
| 3 | 0.965 | 364 | 422 | 490 | 548 | 595 | 639 | 669 | 691 | 705 | 715 | 728 |
| 4 | 0.744 | 288 | 357 | 438 | 505 | 552 | 602 | 630 | 647 | 654 | 645 | 633 |
| 5 | 0.522 | 246 | 319 | 404 | 476 | 537 | 577 | 608 | 629 | 633 | 615 | 591 |
| 6 | 0.301 | 218 | 298 | 391 | 472 | 536 | 587 | 620 | 638 | 640 | 625 | 605 |
| 7 | 0.08 | 170 | 259 | 363 | 456 | 531 | 591 | 631 | 653 | 664 | 644 | 617 |
| 8 | 0 | 123 | 225 | 343 | 439 | 526 | 594 | 636 | 667 | 683 | 660 | 629 |

| Deviation 12 | 1  | -1 | 0  | 0.089 | 0.193 | 0.297 | 0.399 | 0.5 | 0.599 | 0.696 | 0.789 | 0.879 | 1 |
| Final step | 2 | 1  | 19 | 10 | 1 | -2 | 3 | -5 | 0 | 2 | 6 | -2 | -1 |
| 3 | 0.965 | 8 | 5 | 2 | 0 | 2 | 0 | 3 | 4 | 0 | -3 |
| 4 | 0.744 | -2 | 1 | 4 | 2 | 2 | 5 | 4 | 4 | 2 | 3 | 4 |
| 5 | 0.522 | 4 | 3 | 2 | 2 | 3 | 4 | 4 | 2 | 2 | 4 | 5 |
| 6 | 0.301 | 3 | 3 | 2 | 2 | 1 | 1 | 2 | 2 | 3 | 0 | -3 |
| 7 | 0.08 | 5 | 3 | 0 | 1 | 1 | 0 | 3 | 4 | 2 | 1 | 1 |
| 8 | 0 | 8 | 3 | -2 | 1 | 1 | 0 | 3 | 5 | 0 | 2 | 4 |
Table 2. Topological deviations and modifications according to iteration steps (left flank, values in μm)

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| Modification 14 | 1  | -1 | 0 | 0.089 | 0.193 | 0.297 | 0.399 | 0.5 | 0.599 | 0.696 | 0.789 | 0.879 | 1 |
| (Final step) | 2  | 1  |  158 | 241 | 337 | 427 | 506 | 577 | 638 | 682 | 713 | 720 | 730 | |
| 3 | 0.965 | 166 | 246 | 340 | 428 | 506 | 577 | 638 | 682 | 713 | 720 | 730 | |
| 4 | 0.744 | 211 | 279 | 358 | 437 | 509 | 576 | 630 | 673 | 703 | 717 | 736 | |
| 5 | 0.522 | 267 | 325 | 393 | 457 | 516 | 570 | 617 | 656 | 686 | 704 | 728 | |
| 6 | 0.301 | 310 | 360 | 418 | 481 | 533 | 582 | 622 | 658 | 685 | 705 | 732 | |
| 7 | 0.08 | 431 | 465 | 505 | 544 | 583 | 619 | 652 | 682 | 707 | 729 | 759 | |
| 8 | 0 | 548 | 566 | 587 | 610 | 633 | 656 | 680 | 705 | 731 | 756 | 790 | |

| Deviation 15 | 1  | -1 | 0 | 0.089 | 0.193 | 0.297 | 0.399 | 0.5 | 0.599 | 0.696 | 0.789 | 0.879 | 1 |
| (Final step) | 2  | 1  |  158 | 241 | 337 | 427 | 506 | 577 | 638 | 682 | 713 | 720 | 730 | |
| 3 | 0.965 | 0 | 1 | 2 | 1 | 1 | 2 | 3 | 4 | 4 | 1 | -3 | |
| 4 | 0.744 | -1 | 1 | 4 | 2 | 3 | 2 | 2 | 3 | 1 | -2 | |
| 5 | 0.522 | 6 | 4 | 1 | 0 | 2 | 3 | 4 | 5 | 5 | 1 | -4 | |
| 6 | 0.301 | 2 | 3 | 4 | 2 | 2 | 3 | 3 | 4 | 2 | 2 | 1 | |
| 7 | 0.08 | 6 | 3 | 1 | 3 | 4 | 3 | 4 | 5 | 2 | 3 | 3 | |
| 8 | 0 | 11 | 4 | -3 | 5 | 5 | 3 | 5 | 5 | 2 | 4 | 5 | |
Step 14 (Final step)
We could reach the final topological modification after 14 steps for the left flank. You can find the final modification as Modification 14 and the final deviation as Deviation 15. You can see all the deviation values are less than the maximum deviation of 5μm except the values at the border. The graphical comparison of the modification surfaces of Step 1 and the Step 14 (final step) are shown in Figure 8.

Figure 8. Modifications for left flank at Step 1 (left) and Step 14 (right)

4 CONCLUSIONS

The developed method makes it possible to incorporate any desired flank form of a bevel gear given by grid points, and provides the model for the CAM machining in a very short time from the simplest way. That is, the macro geometry is generally assumed by existing standards or data sheets, and the micro-geometry is created by a difference of unmodified real flank to the flank created by topological modifications with the help of KISSsoft. The results showed that the final flank with the topological modification gives the deviation of less than 5μm which can be ignorable considering the manufacturing tolerance in practical situation.

The presented method has considerably high potential for the practical usage, because it allows not only the modeling all existing flank forms into 3D models, but also can also be applied various other purposes, such as to compensate hardening distortions and cutting deviations of 5-axis milling models. These are very important features in practice, and were yet unresolved issues in the 5-axis milling process.

REFERENCES

(1) KISSsoft AG, Calculation program for machine design, http://www.kisssoft.ch/
(5) ISO 21771 (2007) Gears – Cylindrical involute gears and gear pairs – Concepts and geometry